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Master QFin, CTFI
Final Exam, Tue 14.6. 10.00-11.15

Hints

- This is a closed-book exam, one-page cheat sheet is permitted.
- Good luck !

1. Ornstein Uhlenbeck process and Feynman Kac Consider the solution X of the SDE $dX_t = -\kappa X_t dt + dW_t$, $X_0 = x \in \mathbb{R}$ for a standard Brownian motion W and some $\kappa > 0$.

a) (4 points)Show that the process Y with $Y_t = e^{\kappa t} X_t$ has dynamics $dY_t = e^{\kappa t} dW_t$. Conclude that $Y_t = x + \int_0^t e^{\kappa s} dW_s$ and hence

$$X_t = e^{-\kappa t} x + e^{-\kappa t} \int_0^t e^{\kappa s} dW_s.$$

Use this to compute $E(X_t)$.

b) (3 points) Use the Feynman Kac formula to solve the terminal value problems.

$$f_t(t,x) - x f_x(t,x) + \frac{1}{2} f_{xx}(t,x) = 0, \quad (t,x) \in [0,T) \times \mathbb{R},$$

with terminal condition $f(T,x) = x$. Hint: use a) for the computation.

2. Models for asset prices (4 points) Mention two empirical deficiencies of geometric Brownian motion as a model for stock prices. Explain how these deficiencies are reflected in observed option prices.

3. Black Scholes model and binary option. (9 points) Consider in the context of the Black Scholes model with stock price dynamics $dS_t = \mu S_t dt + \sigma S_t dW_t$, initial stock price $S_0 > 0$ and with money market account $B_t = \exp(rt)$ for $r > 0$ a so-called binary option with payoff $h(S_T) = 1_{\{S_T < K\}}$ for some $K > 0$.

a) (1 point) Write down the terminal value problem for the fair price $u(t,S)$ of the option.

b) (3 points) Use the risk neutral pricing formula to compute the price of the option at time $t < T$.

c) (3 points) Compute explicitly the the hedging strategy for the option. Discuss qualitatively potential problems in the implementation of the strategy. Hint: consider the value of the option delta for $S \approx K$ and a small time to maturity.

d) (2 points) Which sign would you expect for the Vega of the option. Distinguish the case where S is substantially lower than K and the case where $S > K$. (An intuitive economic argument is enough, a formal computation is not necessary).